Energy Efficient Estimation of Gaussian Sources Over Inhomogeneous Gaussian MAC Channels

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Abstract— **In this paper, we first provide a joint source and channel coding (JSCC) approach in estimating Gaussian sources over Gaussian MAC channels, as well as its sufficient and necessary condition in restoring Gaussian sources with a prescribed distortion value. An interesting relationship between our proposed joint approach with a more straightforward separate source and channel coding (SSCC) scheme is further established. We then formulate constrained power minimization problems to minimize total transmission power consumption under a distortion constraint for arbitrary in-homogeneous networks under JSCC, SSCC and uncoded scheme (UC). They are transformed to relaxed convex geometric programming problems. Our numerical results exhibit that none of the three schemes is consistently most energy efficient. The proposed JSCC could be more energy efficient than either the uncoded scheme, or SCCC, but not both. In addition, we prove that the optimal decoding order to minimize the total transmission powers for both source and channel coding parts is solely subject to the ordering of MAC channel qualities, and has nothing to do with the ranking of measurement qualities across measuring nodes.**

I. Introduction

In this paper, assuming L sensor nodes send measurements of a Gaussian source to a fusion center via a one-hop interference limited wireless link, we investigate the issue of power allocations across sensors with and without local compression and channel coding. A similar system model for Gaussian sensor networks has also been adopted recently by [1], [2], [3]. In [1], [2], they showed that uncoded transmission achieves the lower bound of the mean squared error distortion as the number of sensors grow to infinity in *symmetric* networks. However, no exact source and channel coding schemes are provided for general system settings other than the uncoded scheme.

In [3], the "exact" optimality of uncoded transmission is proved even for homogeneous Gaussian networks with *finite* number of sensor nodes. As pointed out in [3], it remains unclear though what approach is more favorable when a system becomes non-symmetric with a finite number of sensors.

The objectives of this paper are twofold. First, we will propose a joint source-channel coding approach and then establish its relationship with the separate source and channel coding strategy. Second, we will investigate the optimal rate and power allocation strategy in order to minimize the total transmission power under the constraint that the mean squared error value in estimating the Gaussian source remotely is no greater than a prescribed threshold. In particular, we will compare the resulting total power consumptions of three distinct processing schemes, namely, joint source and channel coding, separate source and channel coding and uncoded amplify-and-forward approaches for in-homogeneous networks, and demonstrate that none of the three schemes is consistently most energy efficient.

II. System Model

Assume L sensor nodes observe a common Gaussian source $X_0[i], i = 1, \cdots, n$, where $X_0[i] \sim \mathcal{N}(0, \sigma_S^2)$ are identically and independently distributed Gaussian random variables with zero mean and variance σ_S^2. The measurements $X_j[i] = X_0[i] + N_j[i],\ j = 1, \cdots, L$ from L sensors experience independent additive Gaussian measurement noise $N_j[i] \sim \mathcal{N}(0, \sigma_{N_j}^2)$, where independence is assumed to hold across both space and time. Let $Y_j[i]$ denote the transmitted signal from sensor j at time i, which satisfies an average power constraint: $\frac{1}{n}\sum_{i=1}^{n} |Y_j[i]|^2 \leq P_j$ for $j = 1 \cdots, L$. The processed signals $\{Y_j[i]\}$ then go through a multiple access channel (MAC) and are superposed at a fusion center resulting in $Z[i] = \sum_{j=1}^{L} \sqrt{g_j} Y_j[i] + W[i]$, where $W[i] \sim \mathcal{N}(0, \sigma_W^2)$ are white Gaussian noise introduced at the fusion center and assumed independent with $N_j[i]$. Coefficients $g_j, j = 1, \cdots, L$ capture the underlying channel pathloss and fading from sensors to the fusion center. In this paper, we assume coherent fusion is conducted in the sense that g_j are assumed perfectly known by the fusion center. Upon receiving $\{Z[i]\}$, the fusion center constructs an estimate $\{\hat{X}_0[i]\}$ of $\{X_0[i]\}$ such that the average mean squared error $D_E \triangleq \lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} E\left|X_0[i] - \hat{X}_0[i]\right|^2$ of the estimation satisfies $D_E \leq D$, where D is a prescribed upper bound for estimation error.

What interests us in this paper is the power efficient schemes to estimate the Gaussian source remotely with a prescribed mean squared error. Three approaches, namely, joint source and channel coding, separate source and channel coding, and uncoded amplify-and-forward schemes, will be investigated in the sequel.

III. Distributed Estimation Approaches over Gaussian MAC Channel

In [4], a joint source and channel coding scheme is proposed to estimate two correlated Gaussian sources remotely

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[4]This project is funded in part from DOE-ORNL (Sensornets Program Sept. 2006- 2008)

978-1-4244-2324-8/08/$25.00 © 2008 IEEE.

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE "GLOBECOM" 2008 proceedings.

at a fusion center where measurements from two sensors are received through a Gaussian MAC channel. Achievable rate region was obtained as a function of the required distortion tuple in restoring two correlated sources. Inspired by their work, we, in this section, will first develop an achievable rate region for our proposed joint source-channel coding (JSCC) approach for any arbitrary network with $L > 1$ sensor nodes and then demonstrate an interesting relationship of JSCC with a separate source and channel coding scheme (SSCC) which is a straightforward combination of the recent findings on the Gaussian source CEO problem [5] and traditional MAC channel coding [6] with independent sources.

A. Joint Source-Channel Coding Approach

Let $\tilde{R}_j, j = 1, \cdots, L$ denote the compression rate at the j-th sensor. There are total $2^{n\tilde{R}_j}$ source codewords $\mathbf{U}_j = \{\mathbf{U^{(k)}}_j, k = 1, \cdots, 2^{n\tilde{R}_j}\}$, for sensor j to represent $\mathbf{X_j} = \{X_j[i], i = 1, \cdots, n\}$. The joint approach we propose here is to let each sensor directly transmit a scaled version of a source codeword $\mathbf{U}_j$. The scaling factor introduced herein is to maintain the average transmission power P_j by sensor j, $j = 1, \cdots, L$. Since L sensors see the same Gaussian source with independent measurement noise, L quantization vectors $\{\mathbf{U_j}, j = 1, \cdots, L\}$ are correlated. As a result, the decoding at fusion center needs to take into account of such correlation when it performs joint decoding of these L codewords. The decoded source/channel codeword $\hat{\mathbf{U}}_\mathbf{j}$ are then linearly combined to obtain an MMSE estimate $\{\hat{X}_0[i], i = 1, \cdots, n\}$ of the Gaussian source $\{X_0[i], i = 1, \cdots, n\}$.

We are interested in deriving the achievable region of rate tuples $\{\tilde{R}_j, j = 1, \cdots, L\}$ such that $2^{n\tilde{R}_j}, j = 1, \cdots, L$ source/channel codewords can be decoded with asymptotic zero error and the mean squared error D_E satisfies $D_E \leq D$.

Lemma 1: To make $D_E \leq D$, $\tilde{R}_i$ satisfy

$$\tilde{R}_i = I(X_i; U_i) = r_i + \frac{1}{2}\log\left[1 + \frac{\sigma_S^2}{\sigma_{N_i}^2}\left(1 - 2^{-2r_i}\right)\right], \quad (1)$$

for $i = 1, \cdots, L$, where where $r_i \geq 0$ are chosen based on

$$\frac{1}{D_E} = \frac{1}{\sigma_S^2} + \sum_{k=1}^{L} \frac{1 - 2^{-2r_k}}{\sigma_{N_k}^2} \geq \frac{1}{D}, \quad (2)$$

and $I(X_j; U_j)$ denotes the mutual information between X_j and a Gaussian random variable U_j, which is associated with X_j by

$$U_j = X_j + V_j, j = 1, \cdots, L \quad (3)$$

where V_j, independent of X_j, are independent Gaussian random variables with mean 0 and variance $\sigma_{V_i}^2 = \sigma_{N_i}^2/(2^{r_i} - 1)$.

Proof: The proof is a straightforward application of the techniques used in proving Lemma 10 in [5]. For brevity, we only provide an outline here.

We quantize $\{X_j[i]\}$ with $2^{n\tilde{R}_j}$ Gaussian vectors $\{\hat{U}_j[i]\}$ such that the source symbol $X_j[i]$ can be constructed from the quantized symbol through a test channel [6]: $X_j[i] = \hat{U}_j[i] + \hat{V}_j[i]$, where $\hat{V}_j[i]$ is a Gaussian random variable with mean zero and variance $2^{-2\tilde{R}_j}\sigma_{X_j}^2$, which is independent of $\hat{U}_j[i] \sim \mathcal{N}\left(0, (1 - 2^{-2\tilde{R}_j})\sigma_{X_j}^2\right)$. Equivalently, we can also represent $\hat{U}_j[i]$ as $\hat{U}_j[i] = \alpha X_j[i] + \tilde{V}_j[i]$, where α is the linear-MMSE estimate coefficient and $\tilde{V}_j[i]$ is the resultant estimation error. By orthogonal principle, we have $\alpha = \frac{\sigma_{\hat{U}_j}^2}{\sigma_{X_j}^2} = \left(1 - 2^{-2\tilde{R}_j}\right)$, $\tilde{V}_j[i]$ is a Gaussian variable independent of $X_j[i]$ with mean zero and variance $2^{-2\tilde{R}_j}\left(1 - 2^{-2\tilde{R}_j}\right)\sigma_{X_j}^2$. Therefore, after normalization, we obtain

$$U_j = \frac{1}{\alpha}\hat{U}_j = X_j + \frac{1}{\alpha}\tilde{V}_j = X_j + V_j \quad (4)$$

where $V_j \sim \mathcal{N}\left(0, \sigma_{X_j}^2/(2^{2\tilde{R}_j} - 1)\right)$. We introduce variables r_j such that $2^{2\tilde{R}_j} - 1 = \left(2^{2r_j} - 1\right)\frac{\sigma_{X_j}^2}{\sigma_{N_j}^2}$, which proves (3). We can also see that r_j is actually the conditional mutual information between X_j and U_j given X_0, i.e. $r_j = I\left(X_j; U_j | X_0\right)$. Since $I(X_j; U_j) = H(U_j) - H(V_j)$, it is then straightforward to show that (1) holds. It is

Given $U_j = X_j + V_j$ and $X_j = X_0 + N_j$, where N_j and V_j are independent, we can construct the LMMSE estimate of X_0 by $\hat{X}_0 = \sum_{j=1}^{L}\beta_j U_j$, where coefficients β_j can be determined again using Orthogonal Principle. Based on Equations (95) and (96) in [5], we obtain the desired result for the mean squared error in (2).

■

From the proof of Lemma 1, it can be seen that U_i and U_j are correlated due to the correlation between X_i and X_j, whose correlation can be captured by $\rho_{i,j}$ the covariance coefficient between X_i and X_j. It can be computed as

$$\rho_{i,j} = \frac{E[X_i X_j]}{\sqrt{E|X_i|^2 E|X_j|^2}} = \frac{\sigma_S^2}{\sqrt{(\sigma_S^2 + \sigma_{N_i}^2)(\sigma_S^2 + \sigma_{N_j}^2)}} \quad (5)$$

The covariance coefficient $\tilde{\rho}_{i,j}$ between U_i and U_j can be computed accordingly as:

$$\tilde{\rho}_{i,j} = \rho_{i,j}\sqrt{(1 - 2^{-2\tilde{R}_i})(1 - 2^{-2\tilde{R}_j})}. \quad (6)$$

For any given subset $S \subseteq \{1, 2, \cdots, L\}$, define vectors $\mathbf{U}(\mathbf{S}) = \left[U_{\pi_1}, \cdots, U_{\pi_{|S|}}\right]$ and $\mathbf{U}(\mathbf{S^c}) = \left[U_{\pi_{|S|+1}}, \cdots, U_{\pi_L}\right]$, where π is an arbitrary ordering of the L indexes. The covariance matrix of $\mathbf{U} = [\mathbf{U}(\mathbf{S}), \mathbf{U}(\mathbf{S^c})]^T$ can thus be decomposed as

$$\mathbf{\Sigma_U} = E\left[\mathbf{U}\mathbf{U}^T\right] = \begin{bmatrix} \mathbf{\Sigma_S} & \mathbf{\Sigma_{S,S^c}} \\ \mathbf{\Sigma_{S^c,S}} & \mathbf{\Sigma_{S^c}} \end{bmatrix}, \quad (7)$$

where $\mathbf{\Sigma_S}$, $\mathbf{\Sigma_{S^c}}$, $\mathbf{\Sigma_{S,S^c}}$ denote the auto- and cross-covariance matrices of $\mathbf{U}(\mathbf{S})$ and $\mathbf{U}(\mathbf{S^c})$. The entries of $\mathbf{\Sigma_U}$ are $(\mathbf{\Sigma_U})_{i,j} = \tilde{\rho}_{i,j}\sigma_{U_i}\sigma_{U_j}$ for $i \neq j$ and $(\mathbf{\Sigma_U})_{i,i} = \sigma_{U_i}^2$, $i, j \in \{1, \cdots, L\}$, where $\tilde{\rho}_{i,j}$ is obtained in (6) and $\sigma_{U_j}^2 = \sigma_{X_j}^2 / \left(1 - 2^{-2\tilde{R}_j}\right)$.

After each sensor maps the observation vector to U_j, an additional scaling factor $\gamma_j = \sqrt{\frac{P_j}{\sigma_{U_j}^2}}$ is imposed on U_j in order to keep the average transmission power of $Y_j[i] = \gamma_j U_j[i]$ as

P_j. The received signal at the fusion center can thus be written as

$$Z[i] = \sum_{j=1}^{L} \gamma_j U_j[i] \sqrt{g_j} + W[i]. \quad (8)$$

Theorem 1: When each sensor performs independent vector quantization and subsequently transmits the resulting scaled quantization vector through a Gaussian MAC channel, to reconstruct the Gaussian source at fusion center with distortion no greater than D, the necessary and sufficient condition is for any subset $S \subseteq \{1, 2, \cdots, L\}$, the following inequality holds

$$I[U(S); X_0|U(S^c)] + \sum_{i \in S} I[U_i; X_i|X_0] \leq I[U(S); Z|U(S^c)] \quad (9)$$

where

$$\text{LHS} = -\frac{1}{2} \log \left[\frac{D_E}{\sigma_S^2} + \sum_{i \in S^c} \frac{D_E}{\sigma_{N_i}^2} \left(1 - 2^{-2r_i}\right) \right] + \sum_{i \in S} r_i \quad (10)$$

and

$$\text{RHS} = \frac{1}{2} \log \left\{ 1 + \frac{1}{\sigma_W^2} \sqrt{\mathbf{g}}(S)^T \mathbf{Q}_{\mathbf{\Sigma_S}} \sqrt{\mathbf{g}}(S) \right\} \quad (11)$$

with $\sqrt{\mathbf{g}}(S)^T = \left[\sqrt{g_i}, i \in S\right]$ and $\mathbf{Q}_{\mathbf{\Sigma_S}} = \mathbf{\Sigma_S} - \mathbf{\Sigma_{S,S^c}} \mathbf{\Sigma_{S^c}}^{-1} \mathbf{\Sigma_{S^c,S}}$.

Proof: From Lemma 1, we know that each sensor finds from $2^{n\tilde{R}_j}$ codewords the closest one $U_j^{(k)} = \{U_j^{(k)}[i]\}$ to the observation vector $\{X_j[i]\}$ and then amplify-and-forwards $\{Y_j[i]\}$ to the fusion center. The decoder applies jointly typical sequence decoding [6] to seek $\{U_j^{(k)}, j = 1, \cdots, L\}$ from L codebooks which are jointly typical with the received vector $\{Z[i]\}$.

WLOG, re-shuffle $2^{n\tilde{R}_j}$ vectors such that $U_j^{(1)}$ is the vector selected by sensor $j \in \{1, \cdots, L\}$. We assume that a subset $U^{(1)}(S^c) = \{U_j^{(1)}, j \in S^c\}$ has been decoded correctly, while its complementary set $U^{(1)}(S) = \{U_j^{(1)}, j \in S\}$ is in error, which implies that the channel decoder at fusion center is in favor of a set of vectors $U^{(k)}(S) = \{U_j^{(k_j)}, k_j \neq 1, j \in S\}$, instead. Next, We will find the upper bound of the probability that $\left(U^{(1)}(S^c), Z\right)$ and $U^{(k)}(S)$ are jointly typical.

The technique to upper-bond this probability is quite similar as the one for MAC channels with independent channel inputs [6, Chap 15.3]. The major difference here is that the channel inputs from L sensors are correlated because of the testing channel model used in independent source coding, i.e. $U_j = X_j + V_j = X_0 + N_j + V_j$, for $j = 1, \cdots, L$.

The upper bound of the probability that $\left(U^{(1)}(S^c), Z\right)$ and $U^{(k)}(S)$ are jointly typical is therefore

$$2^{n(H(U(S),U(S^c),Z)+\epsilon)} 2^{-n(H(U(S^c),Z)-\epsilon)} 2^{-n\sum_{i \in S}(H(U_i)-\epsilon)} \quad (12)$$

$$= \exp_2 \left(-n \left(H(U(S^c), Z) + \sum_{i \in S} H(U_i) - H(U(S), U(S^c), Z) - (|S| + 2)\epsilon \right) \right) \quad (13)$$

where the first term in (12) is the upper bound for the number of jointly typical sequences of $(U(S), U(S^c), Z)$, the second term in (12) is the upper bound of the probability $P\left(U^{(1)}(S^c), Z\right)$ and the last term in (12) is the upper bound of the probability $P\left(U^{(k)}(S)\right)$. The summation in the last term in (12) is due to the independence of codebooks generated by each sensor and the assumption that decoder is in favor of some $U_j^{(k_j)}$ for $k_j \neq 1$ and $j \in S$, which are independent of $U^{(1)}(S)$.

Since we have at most $2^{n\sum_{j \in S} \tilde{R}_j}$ number of sequences to be confused with $U_j^{(1)}, j \in S$, we need

$$\begin{aligned} \sum_{j \in S} \tilde{R}_j &< H(U(S^c), Z) \\ &+ \sum_{i \in S} H(U_i) - H(U(S), U(S^c), Z) - (|S| + 2)\epsilon \\ &= \sum_{i=1}^{|S|-1} I\left(U_{\pi_i}; U_{\pi_{i+1}}^{\pi_{|S|}}\right) + I\left(U(S); U(S^c), Z\right) - (|S| + 2)\epsilon \end{aligned} \quad (14)$$

for all $S \subseteq \{1, \cdots, L\}$ and any arbitrarily small ϵ in order to achieve the asymptotic zero error probability as $n \to \infty$, which thus completes the proof. ■

B. Relationship between JSCC and Separate Source-Channel Coding Approach

If we look closely at (9) and (10), we can easily see that the LHS of the achievable rate region for the JSCC approach actually characterizes the rate-distortion region for Gaussian sources with conditionally independent (CI) condition [5].

Under the CI assumption, distributed source coding at sensors includes two steps. The first step is the same as in JSCC, in which an independent vector quantization for Gaussian source at each sensor is conducted with respect to the observed signal $\mathbf{X_j} = \{X_j[i], i = 1, \cdots, n\}$, which generates a vector $\mathbf{U_j^k} = \{U_j^k[i], i = 1, \cdots, n\}$, $k = 1, \cdots, 2^{n\tilde{R}_j}$. In the second step, those indexes of k_j are further compressed using Slepian-Wolf's random binning approach [5]. Consequently, there are 2^{nR_j} bins for sensor j, which contain all representation vectors $\mathbf{U_j^k}$ of measurements $\mathbf{X_j}$. It was shown in [5] that R_j satisfy: $\sum_{j \in S} R_j \geq I[U(S); X_0|U(S^c)] + \sum_{i \in S} I[U_i; X_i|X_0]$, for all $S \subseteq \{1, 2, \cdots, L\}$ in order to restore X remotely with distortion no greater than D.

For SSCC, to send indexes of bins correctly to the fusion center, independent Gaussian codewords $\{Y_j[i] \sim \mathcal{N}(0, P_j)\}$ for $i = 1, \cdots, n$ and $j = 1, \cdots, L$ for each bin index are generated at L sensors. To ensure indexes are correctly decoded at the fusion center, the rate tuple $\{R_i, i = 1, \cdots, L\}$ should also be contained in the capacity region of Gaussian MAC channel with independent channel inputs under power constraints $\{P_j, j = 1, \cdots, L\}$. The region is characterized by $\sum_{i \in S} R_i \leq \frac{1}{2} \log \left[1 + \sum_{j \in S} \frac{P_j g_j}{\sigma_W^2}\right]$, for $S \subseteq \{1, 2, \cdots, L\}$.

Under SSCC, we can therefore obtain the sufficient and necessary condition for restoring X_0 with MSE no greater than D:

$$I\left[U(S);X_0|U(S^c)\right]+\sum_{i\in S}I\left[U_i;X_i|X_0\right]$$
$$\leq\frac{1}{2}\log\left[1+\sum_{j\in S}\frac{P_jg_j}{\sigma_W^2}\right],\forall S\subseteq\{1,2,\cdots,L\}. \quad (15)$$

In general, we cannot say which approach, JSCC or SSCC, is better in terms of the size of rate region. This can be seen more clearly when we look at a particular case for $L=2$. When there are only two sensors, to reconstruct $\{X_0[i]\}$ with a distortion no greater than D using JSCC or SSCC proposed as above, the transmission powers P_1 and P_2, as well as r_1 and r_2 satisfy:

$$r_1-\frac{1}{2}\log\left\{\frac{D_E}{\sigma_S^2}+\frac{D_E}{\sigma_{N_2}^2}\left(1-2^{-2r_2}\right)\right\}$$
$$\leq\frac{1}{2}\log\left(1+\frac{P_1g_1(1-\tilde{\rho}_{1,2}^2)}{\sigma_W^2}\right) \quad (16)$$

$$r_2-\frac{1}{2}\log\left\{\frac{D_E}{\sigma_S^2}+\frac{D_E}{\sigma_{N_1}^2}\left(1-2^{-2r_1}\right)\right\}$$
$$\leq\frac{1}{2}\log\left(1+\frac{P_2g_2(1-\tilde{\rho}_{1,2}^2)}{\sigma_W^2}\right) \quad (17)$$

$$r_1+r_2+\frac{1}{2}\log\left(\frac{\sigma_S^2}{D_E}\right)$$
$$\leq\frac{1}{2}\log\left(1+\frac{P_2g_2+P_1g_1+2\tilde{\rho}_{1,2}\sqrt{P_1g_1P_2g_2}}{\sigma_W^2}\right) \quad (18)$$

where $\tilde{\rho}_{1,2}$ denotes the covariance coefficient between U_i and U_j, which is determined as in (6) for JSCC with r_i satisfying

$$1/D_E=\frac{1}{\sigma_S^2}+\sum_{k=1}^{2}\frac{1-2^{-2r_k}}{\sigma_{N_k}^2}\geq\frac{1}{D}, \quad (19)$$

and zero for SSCC, respectively.

It can be easily seen from (16)-(18) that inequalities of (16) and (17) under JSCC are dominated by those under SSCC, i.e. $\{P_j,r_j\}$ satisfying (16) and (17) under JSCC also satisfies the corresponding inequalities under SSCC, while the inequality (18) under JSCC dominates that under SSCC.

To compare SSCC and JSCC, we next formulate a constrained optimization problem in which the objective is to minimize the total transmission power of L sensors with a constraint that the distortion in restoring X is no greater than D. For $L=2$, the problem can be stated as

$$\min_{P_i,r_i,i=1,2}P_1+P_2,\ \text{subject to (16)-(18) and (19).} \quad (20)$$

which becomes power/rate allocations for SSCC and JSCC, respectively, for different correlation coefficients $\tilde{\rho}$.

The constrained optimization problems in (20) are non-convex. They can, however, be solved efficiently using standard techniques in convex optimization by transforming the original problems into relaxed convex geometric programming problems [7], [8].

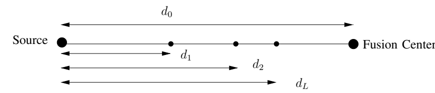


Fig. 1. 1-D location Model

C. Optimal Source/Channel Decoding Order for Non-Symmetric Channels

Although the optimization problems formulated in (20) can only be solved algorithmically, we can still manage to obtain some insights by scrutinizing the problem structures. In this section, we will reveal some relationships between the optimal decoding order and channel attenuation factors for non-symmetric networks.

Theorem 2: Let π^* denote any permutation of $\{1,\cdots,L\}$ such that $g_{\pi^*(1)}\leq g_{\pi^*(2)}\leq\cdots\leq g_{\pi^*(L)}$. To minimize the total transmission power, the optimal decoding order for channel codes at receiver is in the reversed order of π^*, i.e. interference cancellation is in the order $\pi^*(L),\pi^*(L-1),\cdots,\pi^*(1)$, for both SSCC ($L\geq 2$) and JSCC ($L=2$) approaches. The channel decoding order is also the decoding order of distributed source codewords for SSCC.

Proof:

Unlike in the SSCC case where we have a nice geometric (contra-polymatroid) structure [9], which enables us to reach a conclusion valid for any arbitrary asymmetric networks (i.e. $L\geq 2$), JSCC in general lacks such a feature for us to exploit. We can only prove the theorem holds for $L=2$ sensor nodes in the JSCC case [8]. The details can be found in [8]. ■

D. Uncoded Sensor Transmission in Fusion

For uncoded transmission, the transmitted signal by node j is $Y_j[i]=\alpha_jX_j[i]$, where $\alpha_j=\sqrt{\frac{P_j}{\sigma_S^2+\sigma_{N_j}^2}}$ is a scaling factor to make the transmission power $E|Y_j[i]|^2=P_j$. The received signal at the fusion center is therefore $Z[i]=\sum_{j=1}^{L}Y_j[i]\sqrt{g_j}+W[i]$. The linear MMSE estimate of $X_0[i]$ is: $\hat{X}_0[i]=\gamma Z[i]$, where the coefficient γ can be obtained using Orthogonal principle: $E\left[\left(X_0[i]-\hat{X}_0[i]\right)Z[i]\right]=0$. When $L=2$, the power control problem under a distortion constraint $E\left|X_0[i]-\hat{X}_0[i]\right|^2\leq D$ for the uncoded scheme can be similarly formulated and transformed to a GP problem using the condensation technique applied for both SSCC and JSCC [8].

IV. Numerical Results

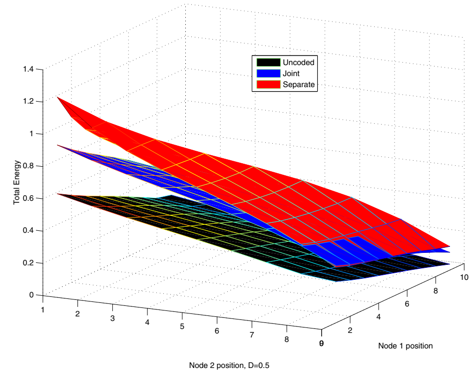
In this section, the three approaches proposed in this paper are examined and compared with each other by looking at each of their optimal total transmission powers under the constraint of restoring X_0 with MSE no greater than D, some prescribed threshold. Particularly, we consider a linear network topology where a source, fusion center and $L=2$ sensor nodes are located on a same line as illustrated in Figure 1. To associate positions of sensor nodes with channel gains and measurement

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE "GLOBECOM" 2008 proceedings.

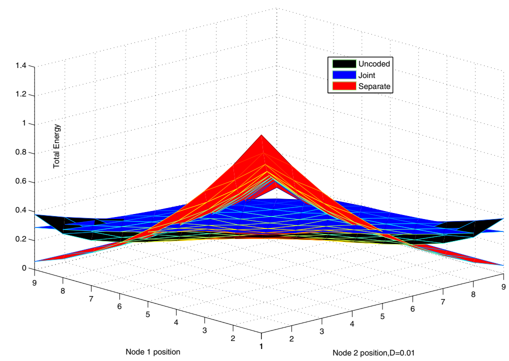
noise, we assume a path-loss model with coefficient β_s and β_c for g_i and $\sigma^2_{N_i}$, respectively: $g_i \propto 1/(d_0 - d_i)^{\beta_c}$ and $\sigma^2_{N_i} \propto d_j^{\beta_s}$, for $i = 1, \cdots, L$, where d_0 is the distance between source and fusion center, and d_i is the distance between the i-th sensor and source. Given a distortion upper-bound $D < \sigma_S^2$ and $\beta_c = \beta_s = \beta$, the distance between the source and fusion center has to satisfy the following inequality, $d_0 < (L)^{1/\beta}\left[\frac{1}{D} - \frac{1}{\sigma_S^2}\right]^{-1/\beta}$, which is obtained by making the MSE using $\{X_i\}$ to estimate X_0 no greater than D.

We then run the geometrical programming based optimization algorithm to determine the minimum total transmission powers for various approaches. We consider 9 spots uniformly distributed between the source and fusion center for possible locations of two sensors, which are indexed by integers 1 through 9. The smaller the index value is, the closer the sensor is located to the source. Figure 2(a), and Figure 2(b) demonstrate the total minimum power consumption $P_1 + P_2$ as a function of nodes' locations for three sensor processing schemes, from which we have following observations:

- When network is symmetric, $P_{total,uncoded} < P_{total,JSCC} < P_{total,SSCC}$.
- Under a relatively large distortion constraint (e.g. $D = 0.5$), uncoded scheme is the most energy efficient among the three candidates for all sensor locations, as shown by Figure 2(a).
- Under relatively small distortion constraints (e.g. $D = 0.01$), separate coding approach becomes the most energy efficient when the relative position of two sensors becomes more asymmetric. For example, in Figure 2(b), at a location with an index pair $(1, 9)$, i.e. the first sensor is closest to the source and the second sensor is closest to the fusion center, we have $P_{total,uncoded} > P_{total,JSCC} > P_{total,SSCC}$.
- Overall, to minimize the total power expenditure, we should choose either uncoded transmission or separate coding scheme for a given pair of locations. This is a bit surprising as joint coded approach is often advocated more efficient (rate wise) than the separate one. It thus exemplifies that exact values of channel conditions and the level of measurement noise are crucial to concluding which scheme is the most power efficient in non-symmetric Gaussian networks with a finite number of sensors.



(a) Result 1: $D = 0.5$, $\sigma_S^2 = \sigma_W^2 = 1$, $\beta_c = \beta_s = 2$.



(b) Results 3: $D = 0.01$, $\sigma_S^2 = \sigma_W^2 = 1$, $\beta_c = \beta_s = 2$
.

Fig. 2. Total power consumption for separate source-channel coding (Red), joint source-channel coding (Blue) and uncoded (Black) schemes

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